Electromagnetic Interactions in a Chiral Effective Lagrangian for Nuclei

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Abstract

Electromagnetic (EM) interactions are incorporated in a recently proposed effective field theory of the nuclear many-body problem. Earlier work with this effective theory exhibited EM couplings that are correct only to lowest order in both the pion fields and the electric charge. The Lorentz-invariant effective field theory contains nucleons, pions, isoscalar scalar (σ) and vector (ω) fields, and isovector vector (ρ) fields. The theory exhibits a nonlinear realization of $SU(2)_L \times SU(2)_R$ chiral symmetry and has three desirable features: it uses the same degrees of freedom to describe the currents and the strong-interaction dynamics, it satisfies the symmetries of the underlying QCD, and its parameters can be calibrated using strong-interaction phenomena, like hadron scattering or the empirical properties of finite nuclei. It has been verified that for normal nuclear systems, the effective lagrangian can be expanded systematically in powers of the meson fields (and their derivatives) and can be truncated reliably after the first few orders. The complete EM lagrangian arising from minimal substitution is derived and shown to possess the residual chiral symmetry of massless, two-flavor QCD with EM interactions. The uniqueness of the minimal EM current is proved, and the properties of the isovector vector and axial-vector currents are discussed, generalizing earlier work. The residual chiral symmetry is maintained in additional (non-minimal) EM couplings expressed as a derivative expansion and in implementing vector meson dominance. The role of chiral anomalies in the EM lagrangian is briefly discussed.

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I. INTRODUCTION

Covariant meson–baryon effective field theories of the nuclear many-body problem (often called *quantum hadrodynamics* or QHD) have been known for many years to provide a realistic description of the bulk properties of nuclear matter and heavy nuclei. (See Refs. [1, 2, 3, 4, 5, 6] for a review.) Recently, a QHD effective field theory (EFT) has been proposed [7, 8, 9, 10, 11] that includes all of the relevant symmetries of the underlying QCD. In particular, the spontaneously broken $SU(2)_L \times SU(2)_R$ chiral symmetry is realized nonlinearly. The motivation for this EFT and illustrations of some calculated results are discussed in Refs. [6, 7, 12, 13, 14, 15, 16, 17]. This QHD EFT has also been applied to a discussion of the isovector axial-vector current in nuclei [18].

One advantage of this QHD EFT is that the electromagnetic (EM) structure of the nucleon and pion can be introduced directly into the lagrangian using a derivative expansion and vector meson dominance (VMD) [7, 19, 20, 21, 22, 23, 24], in addition to the usual "minimal" couplings to the photon field. Nevertheless, the discussion of the EM couplings in Ref. [7] was brief and included only the terms needed for the calculations in that work. In particular, nearly all of the exhibited EM couplings are correct *only to lowest order in both the pion fields and the electromagnetic charge*. The purpose of this work is to illustrate the full structure of the corresponding EM lagrangian, including all terms arising from the introduction of EM gauge-covariant derivatives, and to extend and clarify (and correct) the discussion in Ref. [7]. This full lagrangian will serve as the basis for a calculation of the Lorentz-covariant, one- and two-nucleon amplitudes for both electron scattering and pion photoproduction in the nuclear many-body problem, which will be the subject of a forthcoming publication [25]. It also serves as a launching point for extending the lagrangian to include the Δ resonance [26, 27, 28] as an explicit degree of freedom in the EM interactions.

This QHD EFT has three desirable features: (1) It uses the same degrees of freedom to describe the currents and the strong-interaction dynamics; (2) It respects the same internal symmetries, both discrete and continuous, as the underlying QCD (before and after electromagnetic interactions are included); and (3) Its parameters can be calibrated using strong-interaction phenomena, like πN scattering and the properties of finite nuclei (as opposed to electroweak interactions with nuclei).

There are many, many papers in the literature that discuss EM interactions in the context of chiral, effective, hadronic field theory. (For example, see Refs. [22, 29, 30, 31, 32, 33, 34, 35, 36].) This earlier work has focused almost entirely on EM and radiative meson decays, EM contributions to meson masses, and the EM structure of mesons and nucleons, the latter being carried out primarily in the heavy-baryon formalism [37].

In the present work, we focus on EM interactions in the nuclear many-body problem. We derive the relevant interactions for a Lorentz-invariant QHD lagrangian that contains nucleons and π, σ, ω, and ρ mesons [6, 7]. This lagrangian has a *linear* realization of the $SU(2)_V$ isospin symmetry and a *nonlinear* realization of the spontaneously broken $SU(2)_L \times SU(2)_R$ chiral symmetry (when the pion mass is zero). It was shown in Refs. [7, 8, 11, 38, 39] that by using Georgi's naive dimensional analysis (NDA) [40] and the assumption of *naturalness* (namely, that all appropriately defined, dimensionless couplings are of order unity), it is possible to truncate the lagrangian at terms involving only a few powers of the meson fields and their derivatives, at least for systems at normal nuclear densities [41]. It was also shown that a mean-field approximation to the lagrangian could be interpreted in terms of density functional theory [6, 11, 42, 43, 44], so that calibrating the parameters to observed

bulk and single-particle nuclear properties (approximately) incorporates many-body effects that go beyond Dirac–Hartree theory. Explicit calculations of closed-shell nuclei provided such a calibration and verified the naturalness assumption [8, 38]. *This approach therefore embodies the three desirable features needed for a description of electromagnetic interactions in the nuclear many-body problem.*

We will work in the chiral limit, since the structure of terms involving explicit chiral symmetry breaking is well known [45, 46], and these terms do not change our currents. It is apparent from the lagrangian of two-flavor, massless QCD that when the photon is introduced with the familiar, local $U(1)_Q$ charge symmetry,[1] a residual, global, chiral symmetry remains, which involves left- and right-handed isospin rotations about the "3" axis: $U(1)_{L_3} \times U(1)_{R_3} \times U(1)_B$. (There is also the usual global phase symmetry associated with baryon number B.) This residual global symmetry *must also be present* in the low-energy QHD EFT. We explicitly exhibit this symmetry of our EM lagrangian and discuss how our results are equivalent to the more familiar procedure that uses external sources [45]. [For a lagrangian with only $SU(2)$ isospin and chiral symmetry, as opposed to $SU(3)$, there is no technical advantage to the more formal approach.]

Moreover, we omit the fourth-order pion–pion and pion–nucleon lagrangian $\mathcal{L}_4$, whose structure (and electromagnetic interactions) are also well known [46], since it has not (yet) been relevant in nuclear many-body calculations. Finally, we consider only terms with explicit photon fields. (For a discussion of virtual-photon counterterms, see Refs. [33, 35].)

It is important to note that in our QHD EFT, only pions and nucleons (the hadronically stable particles) can appear on external lines with *timelike* four-momenta. The heavy non-Goldstone bosons appear only on internal lines (with *spacelike* four-momenta) and allow us to parametrize the medium- and short-range parts of the nucleon–nucleon interaction, as well as the electromagnetic form factors of the hadrons using VMD [7, 47]. The heavy bosons are also convenient degrees of freedom for describing nonvanishing expectation values of bilinear nucleon operators, like $\overline{N}N$ and $\overline{N}\gamma^\mu N$, which are important in nuclear many-body systems [1, 6]. Vacuum-loop contributions involving heavy bosons (and nucleons) *are not to be calculated*, since they depend on short-range effects that should be absorbed in the counterterms [48].

The remainder of this paper is organized as follows. Section II briefly reviews the EFT lagrangian of Refs. [7, 18]. Section III defines our decomposition of the EM lagrangian, introduces the EM gauge-covariant derivatives, and derives the (unique) form of the minimal EM current. The residual symmetries of the EM lagrangian and the divergences of the vector and axial-vector currents are also discussed, as is the relationship to the external-field approach. Section IV introduces non-minimal EM interactions in a gradient expansion that is used to describe the pion and nucleon EM structure, and Sec. V discusses the VMD contributions. Section VI contains a brief discussion of the anomalous EM couplings (and other abnormal-parity interaction terms). Section VII is a summary.

[1] For u and d quarks in an isospinor ψ, the coupling to the photon A^μ is $-eA^\mu\overline{\psi}\gamma_\mu Q\psi = -eA^\mu(\overline{\psi}_L\gamma_\mu Q\psi_L + \overline{\psi}_R\gamma_\mu Q\psi_R)$, where $\psi_{R,L} \equiv \frac{1}{2}(1 \pm \gamma_5)\psi$, and the charge matrix is $Q = \frac{1}{2}(\frac{1}{3} + \tau_3)$. Since the chiral symmetry is realized with global rotation matrices, this interaction is invariant under independent left- and right-handed rotations about the third isospin axis.

II. EFFECTIVE FIELD THEORY LAGRANGIAN

The effective field theory (EFT) lagrangian considered in the present work was proposed in Ref. [7]. As discussed in that paper, the nonlinear chiral lagrangian can be organized in increasing powers of the fields and their derivatives. To each interaction term we assign an index

$$\nu \equiv d + \frac{n}{2} + b \,, \tag{1}$$

where d is the number of derivatives, n is the number of nucleon fields, and b is the number of non-Goldstone boson fields in the interaction term. Derivatives on the nucleon fields are not counted in d because they will typically introduce powers of the nucleon mass M, which will not lead to small expansion parameters [7].

It was shown in Refs. [7, 49] that for finite-density applications at and below nuclear matter equilibrium density, one can truncate the effective lagrangian at terms with $\nu \leq 4$. It was also argued that by making suitable definitions of the nucleon and meson fields, it is possible to write the lagrangian in a "canonical" form containing familiar noninteracting terms for all fields, Yukawa couplings between the nucleon and meson fields, and nonlinear meson interactions [50]. See Refs. [6, 7] for a more complete discussion.

If we keep terms with $\nu \leq 4$, the chirally invariant lagrangian can be written as[2]

$$\begin{aligned}
\mathcal{L}_{\rm EFT} &= \mathcal{L}_N + \mathcal{L}_4 + \mathcal{L}_M \\
&= \overline{N}\left(i\gamma^\mu\left[\partial_\mu + iv_\mu + ig_\rho\rho_\mu + ig_v V_\mu\right] + g_A\gamma^\mu\gamma_5 a_\mu - M + g_s\phi\right)N \\
&\quad - \frac{f_\rho g_\rho}{4M}\overline{N}\rho_{\mu\nu}\sigma^{\mu\nu}N - \frac{f_v g_v}{4M}\overline{N}V_{\mu\nu}\sigma^{\mu\nu}N - \frac{\kappa_\pi}{M}\overline{N}v_{\mu\nu}\sigma^{\mu\nu}N + \frac{4\beta_\pi}{M}\overline{N}N\,{\rm Tr}\left(a_\mu a^\mu\right) \\
&\quad + \mathcal{L}_4 + \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{4}f_\pi^2\,{\rm Tr}\left(\partial_\mu U\partial^\mu U^\dagger\right) - \frac{1}{2}{\rm Tr}\left(\rho_{\mu\nu}\rho^{\mu\nu}\right) - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} \\
&\quad - g_{\rho\pi\pi}\frac{2f_\pi^2}{m_\rho^2}{\rm Tr}\left(\rho_{\mu\nu}v^{\mu\nu}\right) + \frac{1}{2}\left(1 + \eta_1\frac{g_s\phi}{M} + \frac{\eta_2}{2}\frac{g_s^2\phi^2}{M^2}\right)m_v^2 V_\mu V^\mu + \frac{1}{4!}\zeta_0 g_v^2 (V_\mu V^\mu)^2 \\
&\quad + \left(1 + \eta_\rho\frac{g_s\phi}{M}\right)m_\rho^2\,{\rm Tr}\left(\rho_\mu\rho^\mu\right) - \left(\frac{1}{2} + \frac{\kappa_3}{3!}\frac{g_s\phi}{M} + \frac{\kappa_4}{4!}\frac{g_s^2\phi^2}{M^2}\right)m_s^2\phi^2,
\end{aligned} \tag{2}$$

where the nucleon, pion, sigma, omega, and rho fields are denoted by N, $\boldsymbol{\pi}$, ϕ, V_μ, and $\rho_\mu \equiv \frac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu$, respectively, with $V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$, and $\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$. The trace "Tr" is in the 2×2 isospin space. The pion field enters through the combinations

$$U \equiv \exp(i\boldsymbol{\tau}\cdot\boldsymbol{\pi}/f_\pi)\,, \qquad \xi \equiv \exp(i\boldsymbol{\tau}\cdot\boldsymbol{\pi}/2f_\pi)\,, \tag{3}$$

$$a_\mu \equiv -\frac{i}{2}\left(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\right)\,, \tag{4}$$

$$v_\mu \equiv -\frac{i}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right)\,, \tag{5}$$

$$v_{\mu\nu} \equiv \partial_\mu v_\nu - \partial_\nu v_\mu + i[v_\mu, v_\nu] = -i[a_\mu, a_\nu]\,. \tag{6}$$

[2] We use the conventions of Refs. [1, 6, 7, 18].

The rho meson enters through the covariant field tensor

$$\rho_{\mu\nu} = D_\mu \rho_\nu - D_\nu \rho_\mu + i\,\overline{g}_\rho [\rho_\mu, \rho_\nu] \ , \tag{7}$$

where the chirally covariant derivative is defined by

$$D_\mu \rho_\nu \equiv \partial_\mu \rho_\nu + i[v_\mu, \rho_\nu] \ , \tag{8}$$

and $\overline{g}_\rho$ is a free parameter [6, 7]. $\mathcal{L}_4$ contains $\pi\pi$ and πN interactions of order $\nu = 4$ that will not be considered further here. (These interactions are exactly the same as in chiral perturbation theory [28, 31, 51].) Numerically small $\nu = 4$ terms proportional to $\phi^2\,\mathrm{Tr}\,(\rho_\mu \rho^\mu)$ and $V_\mu V^\mu\,\mathrm{Tr}\,(\rho_\mu \rho^\mu)$ have been omitted, although they have been considered in Refs. [52, 53].

This EFT lagrangian provides a consistent framework for explicitly calculating the two-body exchange currents originating from meson–nucleon interactions in nuclei. According to naive dimensional analysis (NDA), all of the coupling parameters are written in dimensionless form and should be of order unity, if the theory obeys naturalness; this has been verified for the parameters that are relevant for mean-field nuclear structure calculations [7, 8, 38]. Moreover, all of the constants entering the lagrangian (2) are assumed to be determined from calibrations to nuclear and nucleon structure data, hadronic decays, and πN scattering observables [7, 28].

The lagrangian of Eq. (2) exhibits a nonlinear realization of $SU(2)_L \times SU(2)_R$ chiral symmetry [54, 55]. The transformation properties of the various field combinations have been discussed many times and will not be repeated here. (See, for example, Refs. [7, 11].) Under arbitrary global transformations with matrices $L \in SU(2)_L$ and $R \in SU(2)_R$, the fields are rotated by the local, so-called "compensating-field" matrix $h(x) \in SU(2)_V$, where $SU(2)_V$ is the unbroken vector subgroup of $SU(2)_L \times SU(2)_R$. The matrix $h(x)$ becomes constant only for global $SU(2)_V$ (i.e., isospin) transformations, in which case $L = R = h$.

The full global symmetry $SU(2)_L \times SU(2)_R \times U(1)_B$ implies a conserved baryon current

$$B^\mu = \overline{N} \gamma^\mu N \tag{9}$$

and conserved isovector vector ($\mathbf{V}^\mu$) and axial-vector ($\mathbf{A}^\mu$) currents, which can be determined using Noether's theorem [18, 56]. The isovector currents are given in the Appendix. Noether's theorem also implies that these currents are conserved only for fields that satisfy the Euler–Lagrange equations. The corresponding charges Q^a and Q^a_5 are constants of the motion and satisfy the familiar chiral charge algebra [18].

III. MINIMAL ELECTROMAGNETIC COUPLINGS

The EM interactions will be incorporated by adding to $\mathcal{L}_{\mathrm{EFT}}$ of Eq. (2) the following lagrangian:

$$\mathcal{L}_{\mathrm{EM}} = \mathcal{L}^{\mathrm{min}}_{\mathrm{EM}} + \mathcal{L}^{\mathrm{had}}_{\mathrm{EM}} + \mathcal{L}^{\mathrm{vmd}}_{\mathrm{EM}} + \mathcal{L}^{\mathrm{anom}}_{\mathrm{EM}} \ . \tag{10}$$

The four contributions describe, respectively,

- $\mathcal{L}^{\mathrm{min}}_{\mathrm{EM}}$: terms arising from minimal substitution, obtained by replacing ordinary derivatives in $\mathcal{L}_{\mathrm{EFT}}$ with EM gauge-covariant derivatives (these terms are necessary);
- $\mathcal{L}^{\mathrm{had}}_{\mathrm{EM}}$: non-minimal terms in a derivative expansion, which will serve to describe some of the hadronic EM structure;

- $\mathcal{L}_{\rm EM}^{\rm vmd}$: VMD terms that contain the coupling of the photon to neutral vector mesons (and pions);

- $\mathcal{L}_{\rm EM}^{\rm anom}$: EM terms associated with chiral anomalies, which describe, among other things, mesonic decays like $\pi^0 \to \gamma\gamma$.

In the present section, we will be concerned with the minimal EM couplings, which take the form

$$\mathcal{L}_{\rm EM}^{\rm min} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e A_\mu J^\mu_{\rm min} + \mathcal{L}_{e^2}^{\rm min} \ , \tag{11}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the usual EM field tensor. The terms of $O(e^2)$ contain two factors of the photon field A^μ. Possible terms of higher order in e vanish due to the antisymmetry of the field tensors $v_{\mu\nu}$ and $\rho_{\mu\nu}$. As we will show, the minimal current $J^\mu_{\rm min}$ is conserved through $O(e^0)$.

To include the EM interactions, we elevate a subgroup of the full global symmetry group to the status of a *local* symmetry. This necessitates the introduction of a massless gauge field A_μ and the corresponding gauge-covariant derivatives of the matter fields. The local $U(1)_Q$ symmetry is described by a one-parameter group characterized by a generator ("electric charge")

$$Q = \frac{1}{2} B + T_3 \ , \tag{12}$$

with $T_3 = Q_3$ the third component of isospin and B the baryon number.

Under $U(1)_Q$, the EM field A_μ transforms in the familiar way

$$A_\mu \to A_\mu - \frac{1}{e}\, \partial_\mu \alpha(x) \ . \tag{13}$$

The pion, rho, and nucleon fields transform under a local $U(1)_Q$ rotation in the same fashion as noted earlier, with L, R, and h set equal to

$$q(x) \equiv \exp\left[i\alpha(x) \left(\frac{B + \tau_3}{2} \right) \right] \ , \tag{14}$$

where $B = 0$ for the pion and rho, and $B = 1$ for the nucleon. Thus $N \to qN$, $\xi \to q\xi q^\dagger$, $\rho_\mu \to q\rho_\mu q^\dagger$, etc.

The EM interactions explicitly break the symmetry of $\mathcal{L}_{\rm EFT}$. The lagrangian can be made EM gauge invariant by replacing the ordinary derivatives with the gauge-covariant derivatives [7]

$$\widetilde{\partial}_\mu N \equiv \left[\partial_\mu + \frac{i}{2}\, e A_\mu (1 + \tau_3) \right] N \ , \tag{15}$$

$$\widetilde{\partial}_\mu U \equiv \partial_\mu U + ie A_\mu \left[\frac{\tau_3}{2}, U \right] \ , \tag{16}$$

$$\widetilde{\partial}_\mu \xi \equiv \partial_\mu \xi + ie A_\mu \left[\frac{\tau_3}{2}, \xi \right] \ , \tag{17}$$

$$\widetilde{\partial}_\mu \rho_\nu \equiv \partial_\mu \rho_\nu + ie A_\mu \left[\frac{\tau_3}{2}, \rho_\nu \right] \ , \tag{18}$$

and similarly for the adjoint fields. We will consistently use a *tilde* to distinguish EM gauge-covariant derivatives from ordinary derivatives. As a result of the preceding definitions, the axial and vector pion fields become

$$\widetilde{a}_\mu = a_\mu + \frac{1}{2}\, eA_\mu \left(\xi^\dagger \left[\frac{\tau_3}{2}, \xi\right] - \xi \left[\frac{\tau_3}{2}, \xi^\dagger\right]\right) = a_\mu + \frac{1}{2}\, eA_\mu \left(\xi^\dagger \frac{\tau_3}{2} \xi - \xi \frac{\tau_3}{2} \xi^\dagger\right) \;, \tag{19}$$

$$\widetilde{v}_\mu = v_\mu + \frac{1}{2}\, eA_\mu \left(\xi^\dagger \left[\frac{\tau_3}{2}, \xi\right] + \xi \left[\frac{\tau_3}{2}, \xi^\dagger\right]\right) = v_\mu + \frac{1}{2}\, eA_\mu \left(\xi^\dagger \frac{\tau_3}{2} \xi + \xi \frac{\tau_3}{2} \xi^\dagger - \tau_3\right) \;. \tag{20}$$

[Note that Eq. (19) differs by a minus sign from the expression in Eq. (32) of Ref. [7].] The chirally covariant derivative in Eq. (8) becomes

$$\widetilde{D}_\mu \rho_\nu = D_\mu \rho_\nu + \frac{i}{2}\, eA_\mu \left[\left(\xi \frac{\tau_3}{2} \xi^\dagger + \xi^\dagger \frac{\tau_3}{2} \xi\right), \rho_\nu\right] \;, \tag{21}$$

and the pion field tensor becomes

$$\widetilde{v}_{\mu\nu} = v_{\mu\nu} + \frac{i}{2}\, eA_\mu \left[\left(\xi \frac{\tau_3}{2} \xi^\dagger - \xi^\dagger \frac{\tau_3}{2} \xi\right), a_\nu\right] - \frac{i}{2}\, eA_\nu \left[\left(\xi \frac{\tau_3}{2} \xi^\dagger - \xi^\dagger \frac{\tau_3}{2} \xi\right), a_\mu\right] \;. \tag{22}$$

It is straightforward to verify that the quantities in Eqs. (15) through (22) all transform *homogeneously* under $U(1)_Q$, e.g., $\widetilde{\partial}_\mu N \to q \widetilde{\partial}_\mu N$, $\widetilde{\partial}_\mu \xi \to q(\widetilde{\partial}_\mu \xi) q^\dagger$, $\widetilde{v}_{\mu\nu} \to q \widetilde{v}_{\mu\nu} q^\dagger$, etc.

Using these gauge-covariant derivatives and functions, it is now a straightforward (but tedious) exercise to gauge the original EFT lagrangian and to express the result in the form $\mathcal{L}_{\mathrm{EFT}} + \mathcal{L}_{\mathrm{EM}}^{\mathrm{min}}$ [Eq. (11)], with

$$\begin{aligned}
J^\mu_{\mathrm{min}} = {} & -i\, \frac{f_\pi^2}{4}\, \mathrm{Tr}[\tau_3 (U \partial^\mu U^\dagger + U^\dagger \partial^\mu U)] + \frac{1}{2}\, \overline{N} \gamma^\mu \left(1 + \xi \frac{\tau_3}{2} \xi^\dagger + \xi^\dagger \frac{\tau_3}{2} \xi\right) N \\
& + \frac{1}{2}\, g_A \overline{N} \gamma^\mu \gamma_5 \left(\xi \frac{\tau_3}{2} \xi^\dagger - \xi^\dagger \frac{\tau_3}{2} \xi\right) N + i\, \frac{\kappa_\pi}{M}\, \overline{N} \left[\left(\xi \frac{\tau_3}{2} \xi^\dagger - \xi^\dagger \frac{\tau_3}{2} \xi\right), a_\nu\right] \sigma^{\mu\nu} N \\
& + \frac{4\beta_\pi}{M}\, \overline{N} N\, \mathrm{Tr}\left[\left(\xi \frac{\tau_3}{2} \xi^\dagger - \xi^\dagger \frac{\tau_3}{2} \xi\right) a^\mu\right] + i\, \frac{f_\rho g_\rho}{4M}\, \overline{N} \left[\left(\xi \frac{\tau_3}{2} \xi^\dagger + \xi^\dagger \frac{\tau_3}{2} \xi\right), \rho_\nu\right] \sigma^{\mu\nu} N \\
& + 2 i g_{\rho\pi\pi} \frac{f_\pi^2}{m_\rho^2}\, \mathrm{Tr}\left\{\rho^{\mu\nu} \left[\left(\xi \frac{\tau_3}{2} \xi^\dagger - \xi^\dagger \frac{\tau_3}{2} \xi\right), a_\nu\right] + v^{\mu\nu} \left[\left(\xi \frac{\tau_3}{2} \xi^\dagger + \xi^\dagger \frac{\tau_3}{2} \xi\right), \rho_\nu\right]\right\} \\
& + i\, \mathrm{Tr}\left\{\left[\left(\xi \frac{\tau_3}{2} \xi^\dagger + \xi^\dagger \frac{\tau_3}{2} \xi\right), \rho_\nu\right] \rho^{\mu\nu}\right\} \;,
\end{aligned} \tag{23}$$

$$\begin{aligned}
\mathcal{L}_{e^2}^{\mathrm{min}} = {} & e^2 A_\mu A^\mu\, \frac{f_\pi^2}{4} \left(1 + \frac{4\beta_\pi}{f_\pi^2 M}\, \overline{N} N\right) \left[1 - \frac{1}{2}\, \mathrm{Tr}\left(\tau_3 U \tau_3 U^\dagger\right)\right] \\
& + \frac{e^2}{4} \left(A_\lambda A^\lambda g^{\mu\nu} - A^\mu A^\nu\right) \mathrm{Tr}\left\{\left[\left(\xi \frac{\tau_3}{2} \xi^\dagger + \xi^\dagger \frac{\tau_3}{2} \xi\right), \rho_\mu\right] \left[\left(\xi \frac{\tau_3}{2} \xi^\dagger + \xi^\dagger \frac{\tau_3}{2} \xi\right), \rho_\nu\right]\right\} \\
& + e^2 g_{\rho\pi\pi} \frac{f_\pi^2}{m_\rho^2} \left(A_\lambda A^\lambda g^{\mu\nu} - A^\mu A^\nu\right) \\
& \qquad \times \mathrm{Tr}\left\{\left[\left(\xi \frac{\tau_3}{2} \xi^\dagger + \xi^\dagger \frac{\tau_3}{2} \xi\right), \rho_\mu\right] \left[\left(\xi \frac{\tau_3}{2} \xi^\dagger - \xi^\dagger \frac{\tau_3}{2} \xi\right), a_\nu\right]\right\} \;.
\end{aligned} \tag{24}$$

The EM charge operator can be computed using methods analogous to those in Ref. [18], and it depends only on products of the fields and their conjugate momenta [57]:

$$Q = \int \mathrm{d}^3x \left[\frac{1}{2}\, N^\dagger(1+\tau_3)N + (\boldsymbol{\pi} \,\times\, \boldsymbol{P}_\pi)_3 + (\boldsymbol{\rho}_\nu \,\times\, \boldsymbol{P}^\nu_\rho)_3 \right] \ , \tag{25}$$

where $P^a_\pi \equiv \partial\mathcal{L}/\partial(\partial_0\pi^a)$ and $(P_\rho)^{a\mu} \equiv \partial\mathcal{L}/\partial(\partial_0\rho^a_\mu)$.

There are several relevant observations to be made about the results in Eqs. (23) and (24). Evidently, $J^\mu_{\mathrm{min}} = \frac{1}{2}B^\mu + V^\mu_3$, with B^μ from Eq. (9) and $\mathbf{V}^\mu$ from Eq. (A.1); thus, J^μ_{min} is conserved through $O(e^0)$. The results written in Eqs. (2), (11), (23), and (24) are the most efficient for generating the Feynman rules, since they represent an explicit expansion in powers of the electric charge e. Nevertheless, the individual parts of the lagrangian are *not* EM gauge invariant by themselves; only $\widetilde{\mathcal{L}}_{\mathrm{EFT}} \equiv \mathcal{L}_{\mathrm{EFT}} + \mathcal{L}^{\mathrm{min}}_{\mathrm{EM}}$ is. In particular, the $O(e^2)$ "seagull" terms involving mesons and two photons are crucial for maintaining gauge invariance. Most importantly, although the minimal current of Eq. (23) is conserved through $O(e^0)$, it is not exactly conserved due to the EM interactions: $\partial_\mu J^\mu_{\mathrm{min}} = \partial_\mu V^\mu_3 = O(e) \neq 0$. (The baryon current B^μ remains conserved.) Thus we *cannot* identify J^μ_{min} as the EM current.

To find the conserved, minimal current, we must use Noether's theorem on the gauged lagrangian $\widetilde{\mathcal{L}}_{\mathrm{EFT}}$. A moment of reflection will convince the reader that the desired result can be obtained by simply replacing the ordinary derivatives in J^μ_{min} with gauge-covariant derivatives:

$$\begin{aligned}
\widetilde{J}^\mu_{\mathrm{min}} = & -i\,\frac{f_\pi^2}{4}\,\mathrm{Tr}[\tau_3(U\widetilde{\partial}^\mu U^\dagger + U^\dagger\widetilde{\partial}^\mu U)] + \frac{1}{2}\,\overline{N}\gamma^\mu\left(1+\xi\frac{\tau_3}{2}\xi^\dagger + \xi^\dagger\frac{\tau_3}{2}\xi\right)N \\
& + \frac{1}{2}\,g_A\overline{N}\gamma^\mu\gamma_5\left(\xi\frac{\tau_3}{2}\xi^\dagger - \xi^\dagger\frac{\tau_3}{2}\xi\right)N + i\,\frac{\kappa_\pi}{M}\,\overline{N}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger - \xi^\dagger\frac{\tau_3}{2}\xi\right), a_\nu\right]\sigma^{\mu\nu}N \\
& + \frac{4\beta_\pi}{M}\,\overline{N}N\,\mathrm{Tr}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger - \xi^\dagger\frac{\tau_3}{2}\xi\right)\widetilde{a}^\mu\right] + i\,\frac{f_\rho g_\rho}{4M}\,\overline{N}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger + \xi^\dagger\frac{\tau_3}{2}\xi\right), \rho_\nu\right]\sigma^{\mu\nu}N \\
& + 2ig_{\rho\pi\pi}\frac{f_\pi^2}{m_\rho^2}\,\mathrm{Tr}\left\{\widetilde{\rho}^{\mu\nu}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger - \xi^\dagger\frac{\tau_3}{2}\xi\right), a_\nu\right] + \widetilde{v}^{\mu\nu}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger + \xi^\dagger\frac{\tau_3}{2}\xi\right), \rho_\nu\right]\right\} \\
& + i\,\mathrm{Tr}\left\{\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger + \xi^\dagger\frac{\tau_3}{2}\xi\right), \rho_\nu\right]\widetilde{\rho}^{\mu\nu}\right\} \\
= & \frac{1}{2}B^\mu + \widetilde{V}^\mu_3 \ .
\end{aligned} \tag{26}$$

There are four important things to note about this result. First, the leading-order nucleon terms are the same as in J^μ_{min}, since they contain no derivatives. Second, all factors of τ_3 appear in the combinations $\xi\tau_3\xi^\dagger$ or $\xi^\dagger\tau_3\xi$ (except when they are combined with U). Third, there are no explicit factors of $\widetilde{v}_\mu$; these are all hidden inside the field tensors $\widetilde{\rho}^{\mu\nu}$ and $\widetilde{v}^{\mu\nu}$. Finally, there is no need to use $\widetilde{a}_\mu$ when it is inside a commutator; the $O(e)$ corrections vanish identically by Eq. (19).

To prove that $\widetilde{J}^\mu_{\mathrm{min}}$ is indeed the unique, conserved, minimal EM current, it suffices to evaluate the Euler–Lagrange equations for the photon field based on $\widetilde{\mathcal{L}}_{\mathrm{EFT}}$. One finds

$$\partial_\nu F^{\nu\mu} = e\left(J^\mu_{\mathrm{min}} - \frac{1}{e}\,\frac{\partial\mathcal{L}^{\mathrm{min}}_{e^2}}{\partial A_\mu}\right) = e\,\widetilde{J}^\mu_{\mathrm{min}} \ , \tag{27}$$

where the final equality follows from Eq. (26) and straightforward algebraic manipulation of Eqs. (23) and (24). So we observe immediately that $\widetilde{J}^{\mu}_{\rm min}$ is in fact the source term in Maxwell's equations, and that

$$\partial_\mu \widetilde{J}^{\mu}_{\rm min} = 0 \ , \tag{28}$$

consistent with its identification as the EM current. Note that this current is conserved only for fields that satisfy the Euler–Lagrange equations. Moreover, by adding and subtracting terms containing $(\partial \mathcal{L}^{\rm min}_{e^2}/\partial A_\mu)$, we can rewrite Eq. (11) as

$$\mathcal{L}^{\rm min}_{\rm EM} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - eA_\mu \widetilde{J}^{\mu}_{\rm min} - A_\mu \frac{\partial \mathcal{L}^{\rm min}_{e^2}}{\partial A_\mu} + \mathcal{L}^{\rm min}_{e^2} \ , \tag{29}$$

although the utility of this result is not immediately obvious.

What has become of the isovector currents $\mathbf{V}^\mu$ and $\mathbf{A}^\mu$? It is instructive to begin by studying the residual symmetries of $\widetilde{\mathcal{L}}_{\rm EFT}$. It is clear that with the addition of the EM interactions, the gauged currents $\widetilde{\mathbf{V}}^\mu$ and $\widetilde{\mathbf{A}}^\mu$ are no longer isovectors. Nevertheless, as discussed in the Introduction, massless, two-flavor QCD with EM interactions still possesses a residual, global, chiral symmetry $U(1)_{L_3} \times U(1)_{R_3}$, where the left- and right-handed rotations are around the third axis in isospin space. We now show that this is indeed a symmetry of the gauged lagrangian $\widetilde{\mathcal{L}}_{\rm EFT}$.

For this purpose, it is convenient to consider the lagrangian in the form of Eqs. (2), (11), (23), and (24). The original lagrangian $\mathcal{L}_{\rm EFT}$ is invariant under the full chiral group $SU(2)_L \times SU(2)_R$, so it remains invariant under the residual symmetry. Moreover, the terms in $\mathcal{L}^{\rm min}_{\rm EM}$ involving factors of U and $U^\dagger$ are clearly invariant, since these factors transform globally with the matrices L_3 and R_3, which commute with τ_3. Finally, for left- and right-handed rotations restricted to the third axis in isospin space, we can reduce the original field transformations [7, 11] to

$$\xi(x) \to \xi'(x) = L_3 \xi(x) \widetilde{h}^\dagger(x) = \widetilde{h}(x)\xi(x) R_3^\dagger \ , \tag{30}$$

$$N(x) \to N'(x) = \widetilde{h}(x) N(x) \ , \tag{31}$$

$$\rho_\mu(x) \to \rho'_\mu(x) = \widetilde{h}(x)\rho_\mu(x)\widetilde{h}^\dagger(x) \ . \tag{32}$$

(The photon field is unchanged, as are the isoscalar sigma and omega fields.) Here we use $\widetilde{h}(x)$ to denote local $SU(2)_V$ transformations in the restricted case; note that even though only global rotations L_3 and R_3 are considered, the matrix $\widetilde{h}(x)$ will generally involve isospin rotations in other directions.

We can now make the following observations. First, the nucleon field transforms as in Eq. (31); there are no derivatives of the nucleon field and no explicit factors of v_μ in $\mathcal{L}^{\rm min}_{\rm EM}$. Second, based on Eqs. (30) and (32), all the remaining meson tensors: a_μ, ρ_μ, $\rho_{\mu\nu}$, and $v_{\mu\nu}$ transform *homogeneously*. For example,

$$\rho_{\mu\nu} \to \rho'_{\mu\nu} = \widetilde{h}\rho_{\mu\nu}\widetilde{h}^\dagger \ . \tag{33}$$

All that remains is to examine the pion field combinations

$$\mathcal{Q}_\pm \equiv \xi \frac{\tau_3}{2} \xi^\dagger \pm \xi^\dagger \frac{\tau_3}{2} \xi \ . \tag{34}$$

These are the only combinations of pion fields that have no derivatives, are hermitian, conserve parity, and maintain the residual symmetry. (The parity of $\mathcal{Q}_\pm$ is $\pm$.) The proof of the homogeneous transformation property is simple:

$$\xi^\dagger \tau_3 \xi \;\to\; \xi'^\dagger \tau_3 \xi' = (\widetilde{h} \xi^\dagger L_3^\dagger) \tau_3 (L_3 \xi \widetilde{h}^\dagger) = \widetilde{h} (\xi^\dagger \tau_3 \xi) \widetilde{h}^\dagger \;, \tag{35}$$

$$\xi \tau_3 \xi^\dagger \;\to\; \xi' \tau_3 \xi'^\dagger = (\widetilde{h} \xi R_3^\dagger) \tau_3 (R_3 \xi^\dagger \widetilde{h}^\dagger) = \widetilde{h} (\xi \tau_3 \xi^\dagger) \widetilde{h}^\dagger \;. \tag{36}$$

Note that this proof does not utilize the form of $\widetilde{h}$ and relies only on $[L_3, \tau_3] = 0 = [R_3, \tau_3]$.

The proof of the residual invariance of $\widetilde{\mathcal{L}}_{\rm EFT}$ is now immediate and follows by inspection of Eqs. (23) and (24); $J^\mu_{\rm min}$ and $\mathcal{L}^{\rm min}_{e^2}$ are independently invariant. The gauged lagrangian $\widetilde{\mathcal{L}}_{\rm EFT}$ admits three conserved currents, one of which is B^μ. The other two conserved currents are the gauged neutral currents $\widetilde{V}^\mu_3$ and $\widetilde{A}^\mu_3$; the corresponding charged currents $\widetilde{V}^\mu_\pm$ and $\widetilde{A}^\mu_\pm$ are not conserved. In fact, the explicit result follows from a theorem proven long ago by Adler and Coleman [58], which in our case (and with our notation) reads[3]

$$\partial_\mu \widetilde{V}^\mu_a = \epsilon_{a3b}\, e A_\mu \widetilde{V}^\mu_b \;, \qquad \partial_\mu \widetilde{A}^\mu_a = \epsilon_{a3b}\, e A_\mu \widetilde{A}^\mu_b \;. \tag{37}$$

Note that the theorem does not assume that the field transformations are linear, and it is valid only for fields that satisfy the Euler–Lagrange equations. The divergence of the axial-vector current $\widetilde{\mathbf{A}}^\mu$ omits contributions from chiral anomalies, as well as from the explicit breaking of chiral symmetry. If the latter were included, we would have the PCAC relation

$$\partial_\mu \widetilde{A}^\mu_a \propto m_\pi^2 \pi_a + O(e) \;. \tag{38}$$

Since the charged currents are no longer conserved, the corresponding charges are time dependent. The only constants of the motion are the neutral charges, and the chiral charge algebra reduces to the mutually commuting charges

$$[B, Q_3] = [B, (Q_5)_3] = [Q_3, (Q_5)_3] = [Q, (Q_5)_3] = 0. \tag{39}$$

To end this section, we discuss the relationship between our procedure and the one that uses external fields [45]. In the external-field procedure, one includes EM interactions by introducing spurious charge operators that transform under the full chiral symmetry as

$$Q_L \to Q'_L = L Q_L L^\dagger \;, \qquad Q_R \to Q'_R = R Q_R R^\dagger \;. \tag{40}$$

This permits the construction of so-called "spurion" fields [32, 35] that transform homogeneously under the full symmetry group:

$$\mathcal{Q}_L \equiv \xi^\dagger Q_L \xi \;, \qquad \mathcal{Q}_R \equiv \xi Q_R \xi^\dagger \;, \qquad \mathcal{Q}_{L,R} \to \mathcal{Q}'_{L,R} = h \mathcal{Q}_{L,R} h^\dagger \;. \tag{41}$$

[Compare Eqs. (35) and (36).] One then constructs the most general (non-redundant) lagrangian to a given order in derivatives, and at the end, replaces the spurious charge operators $Q_{L,R}$ with the true charge operator Q to produce a result with the appropriate residual symmetries. Because of the simplicity of the residual symmetry in the present case, it is clear

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[3] An alternative way to write these relations is $\widetilde{\partial}_\mu \widetilde{V}^\mu_a = 0 = \widetilde{\partial}_\mu \widetilde{A}^\mu_a$.

that the appropriate pion field operators (with well-defined parity) are those in Eq. (34). Thus our procedure for constructing $\mathcal{L}_{\rm EM}$ is equivalent to the procedure using external fields.

As a final note, since we have shown earlier that $\mathcal{L}_{e^2}^{\rm min}$ is independently invariant under the residual chiral symmetry, when one computes $O(e^2)$ contributions with photon loops from this lagrangian, one will require additional, *non-minimal* counterterms to render the calculations finite. For instance, if one considers the purely pionic term in Eq. (24), the leading-order (in derivatives) counterterm can be deduced by "integrating out" the photon fields, producing (up to an irrelevant additive constant)

$$\mathcal{L}_{e^2}^{(0)} = e^2\, C\, {\rm Tr}(\tau_3 U \tau_3 U^\dagger)\ . \tag{42}$$

This single counterterm reproduces the well-known $SU(2)$ result [32, 36].

IV. NON-MINIMAL COUPLINGS

Here we discuss couplings of the π and N to the photon that are non-minimal and involve the field tensor $F_{\mu\nu}$ and its derivatives. These terms will be individually EM gauge invariant and are relevant for describing the EM structure of nucleons and pions order-by-order in a derivative expansion [7]. We will consider non-minimal terms only for the hadronically stable particles and return to discuss VMD contributions in the next section.

We begin by considering a generalization of the non-minimal baryon lagrangian proposed by Rusnak and Furnstahl [59]:

$$\begin{aligned}\mathcal{L}_{\rm RF}^{\rm had} = &-\frac{e}{4M}\, F_{\mu\nu} \overline{N}\, \lambda \sigma^{\mu\nu} N - \frac{e}{2M^2}\, (\partial^\nu F_{\mu\nu}) \overline{N}\, \beta \gamma^\mu N \\ &- \frac{e}{4M^3}\, (\partial_\nu \partial^\eta F_{\mu\eta}) \overline{N}\, \lambda' \sigma^{\mu\nu} N - \frac{e}{M^4}\, (\partial^2 \partial^\nu F_{\mu\nu}) \overline{N}\, \beta' \gamma^\mu N + \cdots\ ,\end{aligned} \tag{43}$$

where

$$\lambda = \lambda_{\rm p} \frac{1}{2}\,(1+\tau_3) + \lambda_{\rm n} \frac{1}{2}\,(1-\tau_3) = \frac{1}{2}\,(\lambda_{\rm p}+\lambda_{\rm n}) + \frac{1}{2}\,(\lambda_{\rm p}-\lambda_{\rm n})\tau_3 \equiv \lambda^{(0)} + \lambda^{(1)}\tau_3\ , \tag{44}$$

and similarly for β, λ', and β'. Here the superscripts in parentheses denote the isospin, and $\lambda_{\rm p} = 1.793$ and $\lambda_{\rm n} = -1.913$. The constant λ' contributes to $r_{\rm rms}$ of the anomalous (F_2) EM form factor, and β' contributes to the Q^4 dependence of the charge (F_1) EM form factor. These two terms involve $\nu > 4$ and will be redundant when we include the VMD part of the lagrangian, so we will not consider them further in the sequel.

While Eq. (43) is clearly EM gauge invariant (and baryon phase invariant), it does not obey the residual $U(1)_{L_3} \times U(1)_{R_3}$ chiral symmetry of two-flavor, massless QCD. To maintain this symmetry, we take instead

$$\mathcal{L}_{\rm EM(N)}^{\rm had} = -\frac{e}{4M}\, F_{\mu\nu} \overline{N}\, \widetilde{\lambda} \sigma^{\mu\nu} N - \frac{e}{2M^2}\, (\partial^\nu F_{\mu\nu}) \overline{N}\, \widetilde{\beta} \gamma^\mu N\ , \tag{45}$$

where

$$\widetilde{\lambda} \equiv \lambda^{(0)} + \lambda^{(1)} \left(\xi \frac{\tau_3}{2} \xi^\dagger + \xi^\dagger \frac{\tau_3}{2} \xi \right) = \lambda^{(0)} + \lambda^{(1)} \mathcal{Q}_+\ , \tag{46}$$

and similarly for $\widetilde{\beta}$. The factors $\widetilde{\lambda}$ and $\widetilde{\beta}$ include the appropriate positive-parity combination of pion fields, reduce to the conventional constants when $\boldsymbol{\pi} \to 0$, and contain the fields

that would arise naturally using the external-field construction discussed in the previous section. Evidently, since $[q, \tau_3] = 0$, Eq. (45) remains EM gauge invariant as well, even with ordinary derivatives. Partial differentiation and evaluation of $(\partial \mathcal{L}^{\rm had}_{\rm EM(N)}/\partial A_\mu)$ allows one to identify the contribution to the EM current (in agreement with the source term in Maxwell's equations), which we illustrate below.

We now consider non-minimal couplings between pions and the EM field. These were not discussed explicitly in Ref. [7], since VMD implies that a coupling to rho mesons describes the pion EM form factor quite accurately. We return to this point in the next section.

The lowest-order ($\nu = 4$), non-minimal pion–photon couplings are

$$\mathcal{L}^{\rm had}_{\rm EM(\pi)} = e\,\omega_1 \,{\rm Tr}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger + \xi^\dagger\frac{\tau_3}{2}\xi\right)\widetilde{v}^{\mu\nu}\right]F_{\mu\nu} + e\,\omega_2\,{\rm Tr}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger - \xi^\dagger\frac{\tau_3}{2}\xi\right)\widetilde{a}^{\mu}\right]\partial^\nu F_{\mu\nu}\ . \tag{47}$$

This lagrangian is clearly invariant under the local $U(1)_Q$ charge symmetry. Moreover, since $\widetilde{v}^{\mu\nu}$ and $\widetilde{a}^\mu$ transform homogeneously under the residual chiral symmetry [see Eqs. (19) and (22)], this is also respected.

Partial integration again allows for a determination of the pionic contribution to the non-minimal EM current. When combined with the non-minimal nucleon current discussed above, we find

$$\mathcal{L}^{\rm had}_{\rm EM} = -eA_\mu \widetilde{J}^\mu_{\rm had}\ , \tag{48}$$

$$\begin{aligned}\widetilde{J}^\mu_{\rm had} = {} & \frac{1}{2M}\,\partial_\nu(\overline{N}\,\widetilde{\lambda}\sigma^{\mu\nu}N) - \frac{1}{2M^2}\,\left(g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu\right)(\overline{N}\,\widetilde{\beta}\gamma_\nu N)\\ & - 2\omega_1\,\partial_\nu\,{\rm Tr}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger + \xi^\dagger\frac{\tau_3}{2}\xi\right)\widetilde{v}^{\mu\nu}\right]\\ & + \omega_2\left(g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu\right){\rm Tr}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger - \xi^\dagger\frac{\tau_3}{2}\xi\right)\widetilde{a}_\nu\right]\ .\end{aligned} \tag{49}$$

Note that $\partial_\mu \widetilde{J}^\mu_{\rm had} = 0$ follows by inspection. Moreover, since this relation holds *identically*, it is true whether or not the fields satisfy the Euler–Lagrange equations. Since the baryon current of Eq. (9) remains conserved,

$$\partial^\nu(\overline{N}\,\widetilde{\beta}\gamma_\nu N) = \beta^{(1)}\partial^\nu\left[\overline{N}\,\left(\xi\frac{\tau_3}{2}\xi^\dagger + \xi^\dagger\frac{\tau_3}{2}\xi\right)\gamma_\nu N\right]\ . \tag{50}$$

The lagrangian of Eq. (45) contains no derivatives on the baryon fields, so it gives no new contributions to B^μ, $\widetilde{\mathbf{V}}^\mu$, or $\widetilde{\mathbf{A}}^\mu$. The EM current determined thus far is given by[4]

$$\widetilde{J}^\mu_{\rm min} + \widetilde{J}^\mu_{\rm had} = \frac{1}{2}B^\mu + \widetilde{V}^\mu_3 + \widetilde{J}^\mu_{\rm had}\ . \tag{51}$$

Note that $\widetilde{J}^\mu_{\rm min}$ and $\widetilde{J}^\mu_{\rm had}$ are *independently* conserved, the latter identically and the former by virtue of the Euler–Lagrange equations. Moreover, since the non-minimal terms respect the residual, global chiral symmetry, $\partial_\mu \widetilde{A}^\mu_3 = 0$ remains true as well. The freedom to add $\widetilde{J}^\mu_{\rm had}$,

[4] Although the lagrangian of Eq. (47) generates additional contributions to $\widetilde{\mathbf{V}}^\mu$ and $\widetilde{\mathbf{A}}^\mu$, these are unrelated to the pionic part of $\widetilde{J}^\mu_{\rm had}$, are not identically conserved, and are explicitly of $O(e)$. Thus they are not of particular interest.

which generally contains both isoscalar and isovector parts, reflects the *non-uniqueness* of the EM current in the effective theory. One can always augment the unique minimal current $\widetilde{J}^{\mu}_{\rm min}$ by terms that are independently conserved, without spoiling the conservation of the total EM current. Thus the relationship between the electromagnetic current and $\frac{1}{2}B^{\mu}+V_3^{\mu}$, valid for u and d quarks, is modified in the effective field theory. Note, however, that Q is still given by Eqs. (12) and (25); since $\widetilde{J}^{\mu}_{\rm had}$ is conserved *identically*, it does not produce a new symmetry generator.

V. VECTOR MESON DOMINANCE

There are two basic assumptions underlying the formalism of vector meson dominance:

- Photon interactions with hadrons are mediated primarily by the exchange of low-mass, neutral vector mesons [19, 22].
- One can describe processes involving *spacelike* photons and vector mesons using photon–meson (and meson–meson) couplings determined from hadronic decay widths, in which the meson four-momentum is *timelike*. This hypothesis was justified using dispersion relations in Ref. [20].

For the rho meson, we can start with the expression in Ref. [22]:

$$\mathcal{L}_{\rho} = -\frac{e}{2g_{\gamma}}\,(\partial^{\mu}\rho_3^{\nu} - \partial^{\nu}\rho_3^{\mu})F_{\mu\nu} \ , \tag{52}$$

where g_{γ} (denoted by g_{ρ} in [22]) is determined from $\rho^0 \to e^+e^-$ decay. This expression can be extended to fully reflect the EM gauge invariance and residual chiral symmetry by using

$$\mathcal{L}_{\rho}^{\rm vmd} = -\frac{e}{2g_{\gamma}}\,{\rm Tr}\left[\left(\xi\frac{\tau_3}{2}\xi^{\dagger} + \xi^{\dagger}\frac{\tau_3}{2}\xi\right)\rho^{\mu\nu}\right]F_{\mu\nu} \ . \tag{53}$$

Note that we can use $\rho^{\mu\nu}$ here rather than $\widetilde{\rho}^{\mu\nu}$, since the $O(e)$ corrections produced by the latter vanish identically [see Eq. (21)]. This implies that this VMD term involves only a direct coupling between the photon and rho (and pions), *without any seagulls* involving two photons.

The expression for the $\rho^0 \to e^+e^-$ decay width is [24] (α is the fine-structure constant)

$$\Gamma(\rho^0 \to e^+e^-) = \frac{\alpha^2}{g_{\gamma}^2/4\pi}\left(\frac{m_{\rho}}{3}\right)\left[1+2\left(\frac{m_e}{m_{\rho}}\right)^2\right]\left[1-4\left(\frac{m_e}{m_{\rho}}\right)^2\right]^{1/2} \ . \tag{54}$$

Using the experimental values [60] $\Gamma(\rho^0 \to e^+e^-) = 7.02 \pm 0.11\,{\rm keV}$ and $m_{\rho} = 776\,{\rm MeV}$, we find $g_{\gamma}^2/4\pi = 1.96$, which agrees with Ref. [7].

For the omega meson, we again start with the analysis of Ref. [22] and write

$$\mathcal{L}_0 = \frac{e}{2}\,\frac{1}{2g_Y}\,(\cos\theta_Y\Phi^{\mu\nu} - \sin\theta_Y V^{\mu\nu})\,F_{\mu\nu} \ , \tag{55}$$

where $\Phi^{\mu\nu} = \partial^{\mu}\Phi^{\nu} - \partial^{\nu}\Phi^{\mu}$ and $V^{\mu\nu} = \partial^{\mu}V^{\nu} - \partial^{\nu}V^{\mu}$ are the field tensors for the $\phi(1020)$ and $\omega(782)$, respectively. We will not consider the $\phi(1020)$ coupling further, since it is "integrated out" in our EFT lagrangian, as we discuss below.

Thus we are considering only

$$\mathcal{L}_\omega = -\frac{e}{2}\left(\frac{\sin\theta_Y}{2g_Y}\right) V^{\mu\nu} F_{\mu\nu} \ . \tag{56}$$

As discussed by Sakurai [24], exact $SU(3)_f$ symmetry (denoted by the accent $\circ$) implies that $\mathring{\theta}_Y = 0$, the $\mathring{\phi}$ is pure octet, the $\mathring{\omega}$ is pure singlet, and the hypercharge and isospin couplings are related by

$$\mathring{g}_Y = \frac{\sqrt{3}}{2}\, \mathring{g}_\gamma \ . \tag{57}$$

This relation follows from the $SU(3)$ Clebsch–Gordan coefficients (for pure F-type photon–meson couplings). Thus, to have phi–omega mixing, *there must be explicit* $SU(3)_f$ *symmetry breaking.*

If we assume that the explicit symmetry breaking is described by "ideal mixing", so that the physical $\phi(1020)$ contains only strange (valence) quarks, we find $\sin\theta_Y = 1/\sqrt{3}$. Since this mixing is due to the explicit $SU(3)_f$ symmetry breaking, we can neglect higher-order symmetry-breaking effects by using Eq. (57) to evaluate g_Y, which produces

$$\mathcal{L}_\omega = -\frac{e}{2g_\gamma}\left(\frac{1}{3}\right) V^{\mu\nu} F_{\mu\nu} \ , \tag{58}$$

the result used in Ref. [7].

One can measure θ_Y by comparing the decays $\omega \to e^+e^-$ and $\phi(1020) \to e^+e^-$ [61]. Since the leptonic decays are given by Eq. (54), with appropriate substitution of masses and couplings, one finds

$$\frac{\Gamma(\omega \to e^+e^-)}{\Gamma(\phi \to e^+e^-)} = \frac{m_\omega}{m_\phi}\, \tan^2\theta_Y \ . \tag{59}$$

Using the empirical values [60] $\Gamma(\omega \to e^+e^-) = 0.60 \pm 0.02\,\mathrm{keV}$, $\Gamma(\phi \to e^+e^-) = 1.27 \pm 0.02\,\mathrm{keV}$, $m_\omega = 783\,\mathrm{MeV}$, and $m_\phi = 1019\,\mathrm{MeV}$, we find $(\sin\theta_Y)_{\mathrm{expt}} = 0.617 \pm 0.009$, which differs from the ideal-mixing result $1/\sqrt{3}$ by only 7%. So the real world is close to ideal mixing, and Eq. (58) is adequate for parametrizing hadronic EM form factors.

Thus we take as our VMD lagrangian the sum of Eqs. (53) and (58):

$$\mathcal{L}_{\mathrm{EM}}^{\mathrm{vmd}} = -\frac{e}{2g_\gamma}\left\{ \mathrm{Tr}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger + \xi^\dagger\frac{\tau_3}{2}\xi\right)\rho^{\mu\nu}\right] + \frac{1}{3}\, V^{\mu\nu}\right\} F_{\mu\nu} \ . \tag{60}$$

[Compare Eq. (52) in Ref. [7].] This lagrangian is invariant under the local EM gauge symmetry and the residual, global $U(1)$ chiral symmetry. The factor (1/3) for the $\omega\gamma$ coupling is valid if we assume ideal mixing and keep only the leading-order explicit $SU(3)_f$ symmetry-breaking effects. We note in passing that enforcing the residual chiral symmetry leads to a $\rho\pi\pi\gamma$ coupling that can contribute to a two-nucleon, $\rho\pi$ exchange current in pion photoproduction, in which all the couplings are known from other processes. Vector meson dominance and non-minimal EM couplings were used in Ref. [7] to describe the contributions of nucleon EM structure to the *nuclear* charge form factors, without introducing *ad hoc* form factors at the NNγ vertex.[5] [See Eqs. (56) through (68) in Ref. [7].]

[5] If the ω_i parameters of Eq. (47) are included in the pion charge form factor along with the VMD contribution, a fit to the experimental rms charge radius 0.66 ± 0.01 fm [62, 63] confirms that these parameters are small: $\omega_1 + \omega_2 = (1.2 \pm 0.7) \times 10^{-3}$.

One might argue that in a theory restricted to the light-quark (u, d) sector, arguments based on $SU(3)_f$ symmetry are not particularly relevant. Indeed, we could simply introduce an independent coupling g_0 and take

$$\mathcal{L}'_\omega = -\frac{e}{2g_0} V^{\mu\nu} F_{\mu\nu} \ , \tag{61}$$

analogous to Eq. (52). We can determine g_0 empirically from $\omega \to e^+e^-$ decay, or alternatively, combine this with $\rho^0 \to e^+e^-$ decay to find $g_0^2 = 11.8\, g_\gamma^2$, or

$$g_0 = 3.44\, g_\gamma \ . \tag{62}$$

So we could just use (1/3.44) in place of (1/3) in Eq. (60) to reproduce $\Gamma(\omega \to e^+e^-)$ precisely, but it is remarkable that the estimate based on ideal mixing and lowest-order $SU(3)_f$ symmetry breaking gives such an accurate result. [Interestingly, if we simply set $g_Y = g_\gamma$ in Eq. (56), we would find $g_0^2 = 12 g_\gamma^2$, within 2% of the experimental result!]

With the assumption of ideal mixing, the $\phi(1020)$ is composed only of (valence) strange quarks. Thus it has a weak coupling to nucleons (also true empirically), and its mass is 30% larger than the masses of the ρ and ω. So it is entirely appropriate to integrate out the $\phi(1020)$ degrees of freedom and omit them from the VMD lagrangian (60). Within the explicit $SU(3)_f$ breaking scenario described above, one then finds a contribution to the $\beta^{(0)}$ parameter in $\widetilde{J}^\mu_{\text{had}}$ [Eq. (49)] equal to $-\sqrt{2}M^2 g_\phi / 3 g_\gamma m_\phi^2$, where m_ϕ is the $\phi(1020)$ mass and g_ϕ is its coupling to nucleons. Similarly, the effects of the $\phi(1020)$ in the nucleon–nucleon interaction can be absorbed in the (isoscalar) NNω coupling parameter g_v [7].

As a final comment, we note that it is possible to augment the VMD couplings in Eq. (60) by multiplying the interactions by isoscalar combinations like ϕ, ϕ^2, $V_\mu V^\mu$, etc. (Here ϕ is the field of the σ.) These terms all have $\nu \geq 5$ and contain at least two heavy bosons. Nevertheless, they allow for the possibility of isoscalar EM exchange currents in nuclei.

VI. ANOMALIES

The lagrangian density $\mathcal{L}_{\text{EM}}^{\text{anom}}$ contains two types of terms involving abnormal-parity couplings: terms arising from manifestly chirally invariant expressions (for pions, these first appear at sixth order in derivatives[6] [36, 64]), and terms arising from the Wess–Zumino–Witten action that describes the chiral anomaly [65, 66, 67]. The latter can be expressed as a lagrangian density by using an infinite number of terms that are not chirally invariant [65]; nevertheless, a chiral transformation produces a variation in these terms that is a spacetime derivative, preserving the chiral invariance of the action. All of these abnormal-parity terms contain an odd number of pseudoscalar Goldstone bosons (perhaps coupled to particles with normal parity) and are constrained by G-parity [66, 67]. Overall parity (P) conservation is restored by the presence of the antisymmetric tensor $\epsilon^{\mu\nu\alpha\beta}$ in each term.

Anomalies arise because the fermion measure in the path integral for QCD is not invariant under chiral transformations [68, 69]. This implies that in a quantum field theory with fermions coupled to vector or axial-vector fields, it is generally impossible to satisfy

[6] Here "sixth order" includes external fields, their derivatives, and factors of m_π^2 in the counting. This is equivalent to $\nu = 6$ in our counting scheme.

simultaneously the vector and axial Ward identities derived from the lagrangian through Noether's theorem. The anomalies evidently depend on the fermion couplings to the vector and axial-vector fields, and the form of the anomalous chiral action is well known [67].

Since the anomalies are perturbative, then just as in weak-coupling theories, they can be computed exactly from the underlying QCD. The structure and strength of the anomalies are determined by chiral symmetry and the number of colors N_c in QCD, together with a specific regularization prescription. The results are general and are not unique to the particular low-energy representation in Eq. (2). The contributions from pseudoscalar mesons alone and in combination with vector mesons are discussed, for example, in Refs. [36, 65, 70]. With the power counting used in this work, anomalous interaction terms start at order $\nu = 4$.

EM interactions can be included either by introducing external fields [65] or by using Witten's "trial and error" method [66]. There are terms of $O(e)$ and $O(e^2)$, as required by EM gauge invariance. They can be written using a lagrangian density with a finite number of terms. All the leading-order interaction terms involving pseudoscalar bosons and photons are given by Wess and Zumino [65]; the overall normalization is given in Eq. (21) of Ref. [66]. Applications of the EM anomalous action have focused primarily on meson decays.

For our purposes, the important point is that the anomalous EM terms contain only bosons. Thus they enter in electromagnetic interactions with nuclei only through meson-exchange currents. Moreover, the anomalous EM couplings either: (1) are of $O(e^2)$, like $\pi^0 \to \gamma\gamma$; (2) contain (at least) three pions ($\gamma^* \to \pi\pi\pi$, as in $\omega \to \gamma^* \to \pi\pi\pi$); or (3) involve a heavy boson ($\omega \to \pi^0\gamma$, $\rho \to \pi\gamma$). The resulting abnormal-parity exchange currents are not likely to be very important for studying EM interactions in the nuclear many-body problem, although they may be relevant in few-nucleon systems. (From an examination of the experimental decay widths, the most important contribution probably arises from the $\omega\pi^0\gamma$ coupling.)

Based on these considerations, it is premature to enumerate all of the $\nu = 4$ anomalous EM couplings in this $SU(2) \times SU(2) \times U(1)$ QHD EFT. We relegate this task to future work, when and if it is necessary.

VII. SUMMARY

In this work, we incorporate electromagnetic interactions in a recently proposed hadronic lagrangian with a nonlinear realization of chiral symmetry [7]. The effective lagrangian provides a systematic framework for calculating both nuclear wave functions and nuclear exchange currents. The lagrangian is truncated by working to a fixed order in the parameter ν, which essentially counts powers of ratios of the particle momenta to the nucleon mass M or of mean meson fields to the nucleon mass [7, 59]. Practically speaking, in the nuclear many-body problem, the expansion is in powers of k_{F}/M, where k_{F} is the Fermi wave number at equilibrium nuclear density; this ratio provides a small parameter for ordinary nuclei and for electroweak processes at modest momentum transfers.

For the degrees of freedom considered here (N, π, σ, ω, ρ), truncated at a specific order in terms of fields and their derivatives ($\nu = 4$), we construct the most general (non-redundant) lagrangian consistent with Lorentz invariance; P, C, and T symmetry; electromagnetic gauge invariance; and the residual chiral symmetry of two-flavor, massless QCD with electromagnetic interactions. By introducing EM gauge-covariant derivatives, we formulate the minimal lagrangian required by gauge invariance. This is given by Eqs. (11), (23), and (24), with the EM charge operator of Eq. (25). The conserved current at $O(e^0)$, J^μ_{min}, is equal

to the sum of the Noether currents $\frac{1}{2}B^\mu + V_3^\mu$ of the original lagrangian (V_3^μ is given in the Appendix), but this current is no longer exactly conserved due to the EM interactions. The unique, conserved, minimal EM current, $\widetilde{J}^\mu_{\rm min}$, is obtained by replacing all ordinary derivatives in V_3^μ with gauge-covariant derivatives; it is verified that $\widetilde{J}^\mu_{\rm min}$ [Eq. (26)] is indeed the source in Maxwell's equations.

We show that the gauged neutral currents $\widetilde{V}_3^\mu$ and $\widetilde{A}_3^\mu$ are exactly conserved, but the gauged charged currents $\widetilde{V}_\pm^\mu$ and $\widetilde{A}_\pm^\mu$ are not; that is, $\widetilde{\partial}_\mu \widetilde{V}_a^\mu = 0 = \widetilde{\partial}_\mu \widetilde{A}_a^\mu$. (Here $\widetilde{\partial}_\mu$ is the EM gauge-covariant derivative.) The conserved currents arise because of the residual, global, chiral symmetry $U(1)_{L_3} \times U(1)_{R_3}$, where the left- and right-handed rotations are around the third axis in isospin space. This symmetry obtains in part because the meson charge matrix enters in the form $\mathcal{Q}_\pm$ of Eq. (34), which is identical to the form that would arise in the external-field method of Ref. [45].

Non-minimal couplings of the photon to the nucleon and pion are introduced in Eqs. (45) and (47). These couplings are automatically gauge invariant because they depend on the EM field tensor $F_{\mu\nu}$ and its derivatives, and they respect the residual chiral symmetry because they contain the factors $\mathcal{Q}_\pm$. The enforcement of the residual chiral symmetry implies additional interaction vertices (and meson-exchange currents) containing pions. When combined with the vector-meson-dominance couplings of Eq. (60), these non-minimal couplings are known to adequately reproduce the pion and nucleon EM form factors at low momentum transfers. For the isoscalar ω and $\phi(1020)$ mesons, we assume ideal mixing and keep only the leading-order explicit $SU(3)_f$ symmetry-breaking effects; this reproduces the experimental $\omega \to e^+e^-$ decay width to 30%, which can be easily improved using Eqs. (61) and (62). The EM couplings arising from the anomalous Wess–Zumino–Witten action are also considered, but since these couplings are of order $\nu = 4$ and contribute only in meson-exchange currents, their analysis is left for a future project.

Other future projects based on this QHD EFT lagrangian will include: (1) The computation of the Lorentz-covariant, one- and two-nucleon amplitudes for electron scattering and pion photoproduction (and electroproduction) that can be used in many-body calculations of medium and heavy nuclei [25]; (2) The inclusion of the $\Delta(1232)$ as an explicit degree of freedom and the determination of its EM interactions subject to the residual chiral symmetry mentioned above; and (3) The extension of the non-minimal, hadronic EM lagrangian to include higher-order terms in the derivative expansion [as in Eq. (43)], which will allow the nucleon EM form factors to be accurately described at momentum transfers large enough to study contributions from meson-exchange currents in nuclei.

By working with the QHD EFT lagrangian, which uses the same degrees of freedom to describe the one- and two-body currents and the nuclear many-body dynamics, which respects the underlying symmetries of QCD (both before and after EM interactions are included), and which has parameters that can be calibrated from strong-interaction phenomena (and meson decays), we have a consistent, self-contained, Lorentz-covariant field-theory framework in which to carry out these investigations.

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APPENDIX: ISOVECTOR CURRENTS

For completeness, we list here the expressions for the isovector vector and axial-vector currents originating from the lagrangian (2) to all orders in the pion fields. These expressions generalize Eqs. (142) and (143) of Ref. [18].

$$
\begin{aligned}
V_a^\mu = & -i\,\frac{f_\pi^2}{4}\,\mathrm{Tr}[\tau_a(U\partial^\mu U^\dagger + U^\dagger\partial^\mu U)] + \frac{1}{2}\,\overline{N}\gamma^\mu\left(\xi\frac{\tau_a}{2}\xi^\dagger + \xi^\dagger\frac{\tau_a}{2}\xi\right)N \\
& + \frac{1}{2}\,g_A\overline{N}\gamma^\mu\gamma_5\left(\xi\frac{\tau_a}{2}\xi^\dagger - \xi^\dagger\frac{\tau_a}{2}\xi\right)N + i\,\frac{\kappa_\pi}{M}\,\overline{N}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger - \xi^\dagger\frac{\tau_a}{2}\xi\right), a_\nu\right]\sigma^{\mu\nu}N \\
& + \frac{4\beta_\pi}{M}\,\overline{N}N\,\mathrm{Tr}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger - \xi^\dagger\frac{\tau_a}{2}\xi\right)a^\mu\right] + i\,\frac{f_\rho g_\rho}{4M}\,\overline{N}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger + \xi^\dagger\frac{\tau_a}{2}\xi\right), \rho_\nu\right]\sigma^{\mu\nu}N \\
& + 2ig_{\rho\pi\pi}\frac{f_\pi^2}{m_\rho^2}\,\mathrm{Tr}\left\{\rho^{\mu\nu}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger - \xi^\dagger\frac{\tau_a}{2}\xi\right), a_\nu\right] + v^{\mu\nu}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger + \xi^\dagger\frac{\tau_a}{2}\xi\right), \rho_\nu\right]\right\} \\
& + i\,\mathrm{Tr}\left\{\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger + \xi^\dagger\frac{\tau_a}{2}\xi\right), \rho_\nu\right]\rho^{\mu\nu}\right\}\ ,
\end{aligned}
\tag{A.1}
$$

$$
\begin{aligned}
A_a^\mu = & -i\,\frac{f_\pi^2}{4}\,\mathrm{Tr}[\tau_a(U\partial^\mu U^\dagger - U^\dagger\partial^\mu U)] - \frac{1}{2}\,\overline{N}\gamma^\mu\left(\xi\frac{\tau_a}{2}\xi^\dagger - \xi^\dagger\frac{\tau_a}{2}\xi\right)N \\
& - \frac{1}{2}\,g_A\overline{N}\gamma^\mu\gamma_5\left(\xi\frac{\tau_a}{2}\xi^\dagger + \xi^\dagger\frac{\tau_a}{2}\xi\right)N - i\,\frac{\kappa_\pi}{M}\,\overline{N}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger + \xi^\dagger\frac{\tau_a}{2}\xi\right), a_\nu\right]\sigma^{\mu\nu}N \\
& - \frac{4\beta_\pi}{M}\,\overline{N}N\,\mathrm{Tr}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger + \xi^\dagger\frac{\tau_a}{2}\xi\right)a^\mu\right] - i\,\frac{f_\rho g_\rho}{4M}\,\overline{N}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger - \xi^\dagger\frac{\tau_a}{2}\xi\right), \rho_\nu\right]\sigma^{\mu\nu}N \\
& - 2ig_{\rho\pi\pi}\frac{f_\pi^2}{m_\rho^2}\,\mathrm{Tr}\left\{\rho^{\mu\nu}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger + \xi^\dagger\frac{\tau_a}{2}\xi\right), a_\nu\right] + v^{\mu\nu}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger - \xi^\dagger\frac{\tau_a}{2}\xi\right), \rho_\nu\right]\right\} \\
& - i\,\mathrm{Tr}\left\{\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger - \xi^\dagger\frac{\tau_a}{2}\xi\right), \rho_\nu\right]\rho^{\mu\nu}\right\}\ .
\end{aligned}
\tag{A.2}
$$

Note that the sign of the first term in Eq. (107) [and Eq. (110)] of Ref. [18] is incorrect. This propagates into an additional $\overline{N}N\pi\rho$ coupling in the axial-vector current of Eq. (143), namely,

$$
A_{\text{new}}^{a\,\lambda} = \frac{1}{f_\pi}\,\frac{f_\rho g_\rho}{4M}\,\epsilon^{abc}\epsilon^{cdf}\pi^b\rho_\nu^d\,\overline{N}\sigma^{\lambda\nu}\tau^f N\ . \tag{A.3}
$$

This additional term affects neither the charge algebra nor the tree-level, two-nucleon amplitudes of that paper.

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